A consistent approach to teaching the use of an empty number line for addition and subtraction

There is no hierarchy in calculation methods. Pupils are taught to use the most efficient method for the calculation, for example it would not be efficient to use a formal written method for 2000-1997.

All pupils are taught to use an Empty Number Line (ENL)
By the end of the year 3 pupils should be able to-
* Use extended Number bonds for splitting 1000
* Make valid choices as to appropriate methods to use for given numerical problems, and justify these choices
* Add or subtract mentally pairs of two-digit whole numbers (e.g. 47 + 58, 91 – 35)
* Use strategy labels when discussing their methods

When pupils can confidently do the above they will be taught formal written methods.
Pupils will use the Empty Number Line method as part of a calculation 'toolbox', choosing the most efficient method for the calculation.

Prerequisite competences in order to use the ENL for addition:

The ability to position numbers in the correct order on a number line

Knowledge of all pairs of numbers with a total of ten.

The ability to jump forward and backwards on the number line in jumps other than 1

The ability to instantly add a single digit number to a multiple of ten

An ability to jump on or back in tens from a given number.

Know number splits to 10

Addition using the empty number line

We teach three main strategies for addition.

J10 – jumping in tens
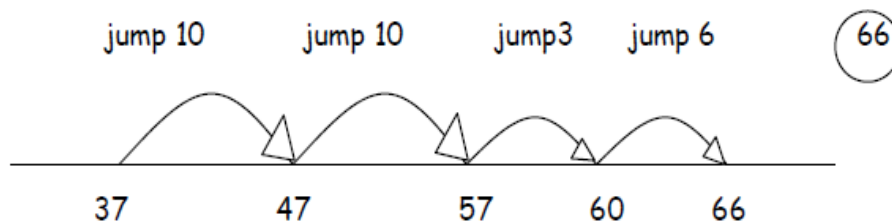
OJ- overjumping

H10- hit the tens

The three methods are demonstrates using the calculation 37+29

J10- The 'Tens jumping' method
This first method is the simplest mental image and could be described as the 'tens jumping' method using the empty number line.
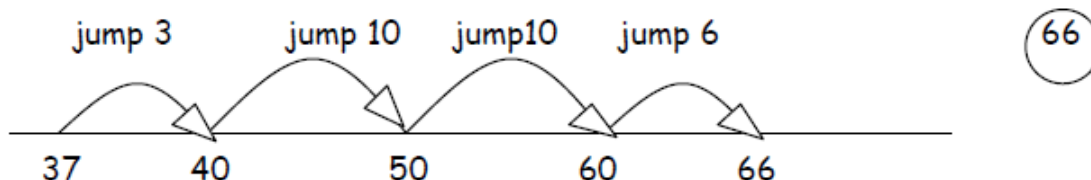
37+29

jump 10   jump 10   jump3   jump 6   66

37   47   57   60   66

H10-Hit the tens
This method involves a first jump to the next multiple of ten, to find a 'comfortable resting place', making the rest of their jumps much simpler.
37 + 29 =66

jump 3   jump 10   jump10   jump 6   66

37   40   50   60   66

As children become more confident with this method they will become increasingly efficient and reduce the number of steps, performing a single jump of several multiples of ten from a "tens number" or "comfortable resting place":

jump 3   jump 20   jump 6   66

37   40   60   66
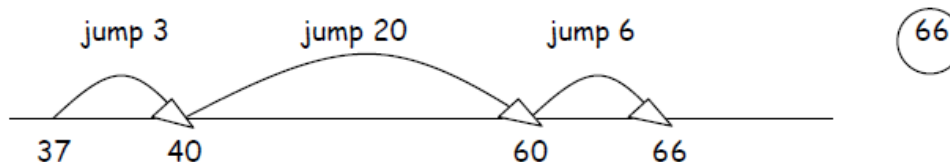
OJ- Over jumping

Pupils round one number up to the nearest multiple of 10 and make an 'overjump', then compensate for the 'over jump' by jumping backwards.

This "overjumping" method is particularly suited to adding numbers like 29 which are close to the next multiple of ten.

For example in 37+29*

*pupils round the 29 to 30
  ➤ make a first jump of 10 from 37 to 47
  ➤ a second jump of 10 from 47to 57
  ➤ a third jump of 10 from 57 to 67, making a total jump of 30
  ➤ and finally jump back 1 to compensate, because pupils are actually adding 29
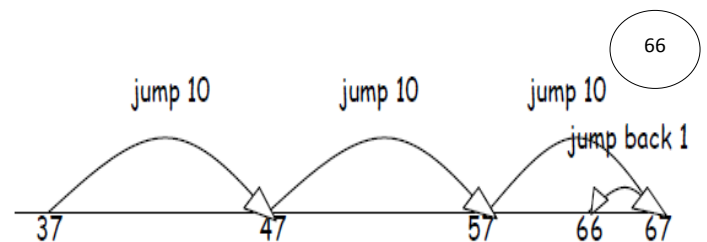
As pupils become more confident they will combine three single jumps of 10 into one jump of 30.
  ✶ pupils round the 29 to 30
  ➤ make a first jump of30,
  ➤ and jump back 1 to compensate, because pupils are actually adding 29.

3
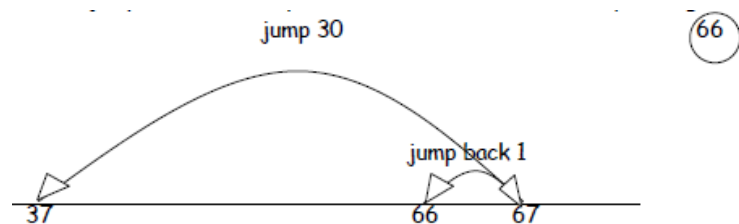
Subtraction using the empty number line.

Subtraction is taught as both

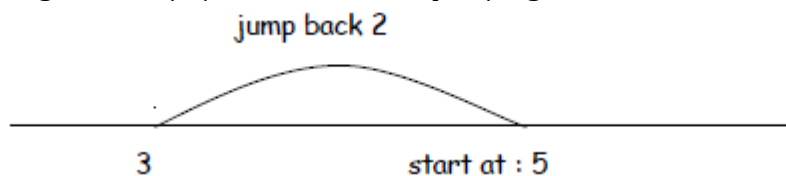**taking away, counting back** from the larger number
and
as difference, counting on from the smaller number

Subtraction as taking away, counting back from the larger number
5-2=3
If I have 5 counters and take 2 away, how many are left? Three.
Using the empty number line " jumping" method:

jump back 2

3   start at : 5

and **as difference counting on** from the smaller number.(Complementary addition)

Children need to understand that 5-2 can also mean:
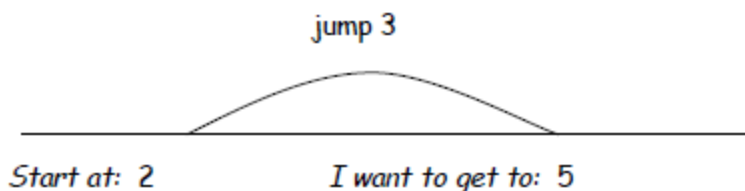What is the difference between 5 and 2?
This can be illustrated by making a tower of 5 cubes, and a tower of 2 cubes.

When we compare the two towers, there is a difference of 3 cubes in their height.

We can illustrate this on the empty number line by thinking of 5-2
as:

jump 3

Start at: 2   I want to get to: 5

what must I add to 2 to get to 5?
Or: If I start at 2, how many do I need to count on to get to 5?

An understanding that both of these simple examples are models for subtraction can enable best use to be made of the complementary addition method for subtraction , which is the fourth method for subtraction using the empty number line.

Methods for subtraction <u>as taking away</u> using the empty number line

Children first learn about subtraction as "taking away.

The same three empty number line methods are used for subtraction <u>as taking away</u> as for addition

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J10 – jumping in tens
OJ- overjumping
H10- hit the tens

Prerequisite competences in order to use the ENL for subtraction <u>as taking away:</u>

- *The ability to position numbers in the correct order*
- *A knowledge of all pairs of numbers with a total of ten.*
- **The ability to jump** *forward and* **backwards on the number line in jumps other than 1**
- *The ability to instantly add a single digit number to a multiple of ten*
- *An ability to jump on or back in tens from a given number.*

The example calculation 43 – 28 = 15 is the type which cause problems for pupils who have rote learnt decomposition
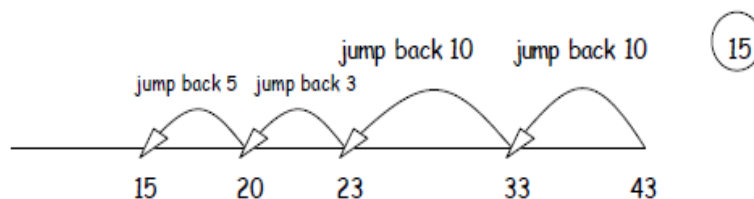
J10- The 'Tens jumping' method

43 – 28 = 15
In the J10 method the calculation can be equated with taking away, which is how children first experience subtraction.

➢ back 10 from 43 to 33
➢ back 10 from 33 to 23
➢ back 3 to 20 (A H10 move to land on a comfortable resting place)
➢ and a final jump back of 5 to 15.

The method involves the pupil seeing 28 as 10+10 +3+5

H10- The 'Hit the tens' method

43 – 28 = 15

In this example an initial jump back of 3 to 40, (A H10 move to find a comfortable resting place.) to hit the first multiple of ten below 43. Once on a tens number, pupils have a 'comfortable resting place' which can enable single jumps of several multiples of ten, in this case a single jump back of 20 - initially pupils will complete this is two jumps of 10. Pupils then have a final jump back of 5

- ➤ initial jump back of 3 to 40
- ➤ jump back 20 to 20
- ➤ jump back 5 to 15
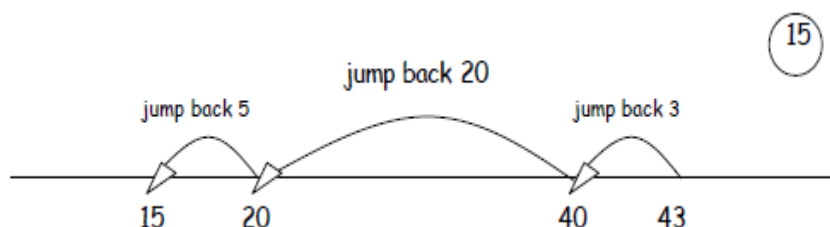
The 'Overjumping' method

43 – 28 = 15

This method involves jumping back too far, in this case making a jump back of 30 to 13, then compensating by jumping forward 2 to 15. This example shows a single backward 'overjump' of 30, initially this will be performed as three backward jumps of 10 but pupils should move to the more efficient method of one jump of 30.
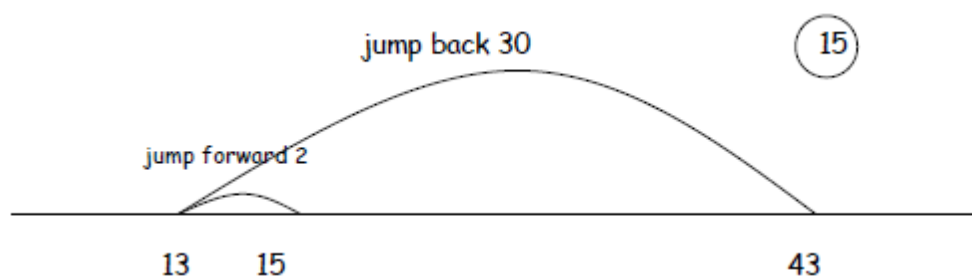
Methods for subtraction <u>difference</u> – Counting on

Using the empty number line for complementary addition

This method for subtraction can be described as the "count-on" method, and its power lies in the premise that pupils find it is easier to count forwards than backwards.

The "count-on" method for subtraction depends on finding the difference between two numbers by "counting on" from one to the other. This "counting on" can be performed efficiently by jumps along the empty number line.

For example

43 – 28 = 15

Using the "count-on" method 28 and then 43 are placed on the empty number line.   The difference between them is found by jumping from the 28 to the 43

> jump from 28 to 30 (a H10 move to find a comfortable resting place)
> jump 10 from 30 to 40
> jump 3 from 40 to 43
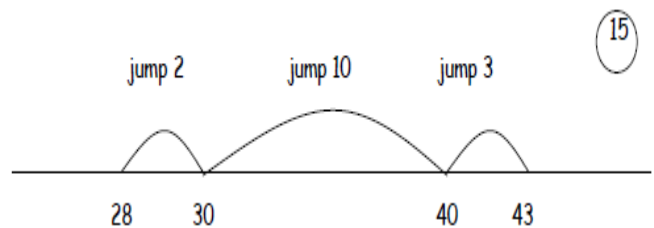
Using this method, the "answer", 15 is given by the sum of the jumps along the number line. This differs from the other methods for both addition and subtraction in which the "answer" is given by the position in which we land on the number line after our final jump.